1. Introduction

At *Constância Investimentos* we base the construction of our portfolios on a systematic approach using traditional risk factors such as Value, Quality, etc. Please see our previous article "*Stock Market Investing through Risk Factors*"[1].

For a long time we used a calendar rebalance criterion, whereby we would fully rebalance at the end of each month towards the most recent allocation suggested by our multifactor model. Monthly rebalance is a common practice in the financial market because it tends reasonably balance and both excessive transaction costs and adverse lag or discrepancy between the current portfolio and the exposures suggested by our model, which change slowly but daily.

However, we had no economic or financial rationale to guide us on whether it was preferable to rebalance more or less frequently than monthly, totally or partially. A more rational rebalance scheme should increase the frequency in times of more volatile markets and possibly when companies are reporting results that differ significantly from analysts' projections. In both situations, the exposures suggested by our model would potentially change more significantly and more quickly, potentially justifying paying transaction costs more often.

In systematic strategies such as the one adopted at Constância Investimentos, where a significant portion of the portfolio may have low liquidity, the transaction cost involves not only brokerage and fees, but also "Market Impact", that is, how much the asset's price would change if the transaction were made. See our article "*Market Impact and Scalability*"[2].

In this article we explain and adapt the theory of optimal rebalancing of portfolios with transaction costs to develop and implement a rational strategy for reallocating our assets under management. As a result, we'll have a portfolio with a much lower turnover and better performance in the back-test after transaction costs. This significantly increases the capacity of our systematic strategy applied to the Brazilian equity market.

[1] https://constanciainvest.com.br/en/publications/
[2] Same link as above

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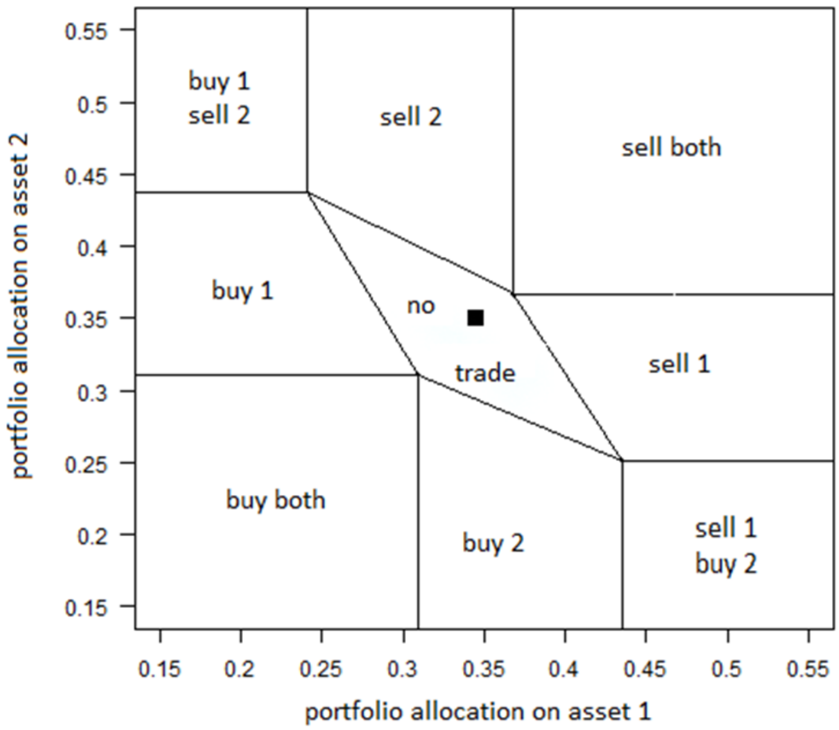
2. Optimal portfolio rebalancing theory.

The topic of optimal rebalancing of portfolios is quite complex and has been the subject of studies in several academic and applied works, see for example references [3], [4] and [5]. All these studies point out that, because of the existence of transaction costs, the strategy usually adopted to fully rebalance towards the ideal portfolio is never an optimal procedure. When balancing transaction costs and potential improvement in the portfolio profile, a partial rebalancing is generally more appropriate.

In fact, these studies prove that there is a "no trading" region around the optimal portfolio. That means if the pre-rebalancing portfolio is close enough to the ideal portfolio, no fractional rebalancing would justify paying the transaction costs, and the optimal strategy would be to do nothing. When there are only proportional costs (brokerage, fees, etc.), the authors prove that an optimal strategy is to rebalance to the border of the "no trading" region. Fixed costs (registration costs, etc.) or mixed fixed/proportional costs induce a rebalancing to the interior of that region.

For a portfolio of only 2 assets, the plot below displays how a racional rebalancing towards the central point would typically work:



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3. Market impact as transaction costs.

The most common market impact model is BARRA's "square root model" (see reference [1]). An order of size Q for a stock with average daily volume V and daily volatility σ, has a percentage price impact $\Delta(Q)$ given by the formula $\Delta(Q) = \alpha\left(\frac{Q}{V}\right)^{\delta}\sigma$, where α and δ are constants estimated by regressions. Using high frequency databases encompassing hundreds of thousands or even millions of trades from US stock market, several econometric work (see references [1] and [2]) have estimated $\alpha \cong 1$ and $\delta \cong 1/2$.

The percentage transaction cost due to the market impact is related to the average execution price, which we assume to suffer half the impact above, that is, $\frac{1}{2}\sqrt{\frac{Q}{V}}\sigma$. Therefore, the final transaction cost is the order size multiplied by that percentage cost $=\frac{1}{2}\sqrt{\frac{Q}{V}}Q\sigma$. Hence, the transaction cost grows at a rate higher than linear, with the exponent 3/2, and may be much higher than typical proportional transaction costs from brokerage and fees, particularly in illiquid stocks where the average daily volume V is low.

In a recent econometric study (see [9]) and which has one of the largest databases with 5 million trades, the authors conclude that the square root model can be excessively conservative for large trades (up to close to 1 daily volume), and a functional form with δ substantially smaller than ½ or even with logarithmic functional form may be more suitable.

As mentioned, the literature suggests a "no trade" region around the target portfolio. Linear transactions costs induce a partial rebalancing up to the border of this region, while fixed costs to within this region. It is possible to show that with the transaction costs due to the Market Impact (of the order of 3/2), this frontier will never be reached. Nonetheless, if the attractiveness of assets (expected return and volatilities) remains unchanged, then it can be demonstrated that successive optimal rebalancing will make exposures asymptotically approach that frontier.

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4. Implementation, b*acktest* and strategy *capacity*

For each stock *i,* let s_t^i be the score generated by our multifactor model The scores and the stock exposures would coincide we could had chosen to ignore the important transaction cost associated with the market impact. The back test performance of this fictional, transaction cost free portfolio, is reported first numerical column "original model" in the table below.

The transaction costs associated with market impacts tend to be prohibitive for less liquid stocks when the strategy's AUM (asset under management) is high. Therefore, in previous article[3], we developed a methodology where we adjusted the exposure suggested by the score s_t^i by limiting the number of daily volumes allowed for any stock to a maximum. The excess exposure from the illiquid stocks that were capped at the maximum are redirected to the more liquid ones. We denote these adjusted exposures suggested by our systematic model as $\alpha_t^{target,i}$. The back test performance results of these adjusted exposures are reported in the second numerical column of the table below.

Let α_t^i be the stock exposure effectively implemented. The monthly rebalance that we adopted for several years did not rely on any economic rationale. It was only a calendar criterium where we used to fully rebalance by setting $\alpha_t^i = \alpha_t^{target,i}$.

The rational rebalance of our long-only portfolio allow for partial rebalance and requires numerical optimizations where we maximize the increase in the expected return, minus the proportional costs (brokerage + fees, which we assume to be 15bps) and the costs of market impact. The optimization gives us the percentage that each stock should be rebalanced towards $\alpha_t^{target,i}$, which can be zero, 100% or a fraction of it. It is important to note that in the rational rebalancing the allocation suggested by our systematic model becomes a "moving target" that will never be achieved.

We rerun this optimization for each end of the month over the history of our back-test, from the beginning of August 2004 to the end of March 2020, and

[3] "*Market Impact and Scalability*". https://constanciainvest.com.br/en/publications/

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compare below the expected returns for the systematic long-only portfolio. As a reference, the average annual return of Ibovespa in this period was 7.9%.

AUM (R$ bi)		Original Model	Monthly Full Rebalance with Exposure capped by Market Impact	Optimized Rebalance
	Gross Return	25.6%	24.3%	21.5%
1	Transaction Costs	10.2%	5.7%	2.2%
	Net Return	15.4%	18.6%	19.3%
	Gross Return	25.6%	22.7%	19.7%
3	Transaction Costs	17.6%	8.4%	3.1%
	Net Return	8.0%	14.3%	16.6%
	Gross Return	25.6%	21.2%	18.5%
5	Transaction Costs	22.0%	11.4%	3.6%
	Net Return	3.6%	9.8%	14.9%
	Gross Return	25.6%	19.8%	16.6%
10	Transaction Costs	32.2%	17.8%	4.6%
	Net Return	-6.6%	2.0%	12.0%

In the table above, we have progressively increased the total AUM of the strategy from R $ 1 billion to R $ 10 billion. The historical average gross backtest return of our original model is 25.6% per year in the period analyzed, regardless of the AUM. Compared to the Ibovespa return of 7.9% p.a, this gives a good measure of the value added by our systematic multifactorial model. However, because a significant portion of the allocations generated by the model are Mid Caps and Small Caps, the estimated transaction cost for an AUM of R $ 1 billion is already quite high at 10.2%. Most of this cost comes from the market impact.

As we mentioned, precisely because of this high cost of market impact, we developed in previous work a methodology based on the market impact theory, where we rationally impose a maximum limit on the number of daily volumes for the exposure of any stock. The surplus of the illiquid stocks with limited exposures are redirected towards other more liquid stocks with a high score.

The back-test results obtained when we use this methodology that limits illiquid exposures but fully rebalance on the last business day of each month are in the second column of the table. For the AUM of R $ 1 billion, this imposition of a maximum number of daily volumes for illiquid shares and reallocation of exposures to more liquid shares reduces the strategy's gross return from 25.6% per year to 24.3%. However, the reduction in transaction costs due to the greater contraction in more liquid stocks is more significant, from 10.2% to 5.7%. Therefore, the average net return on the strategy over the back-test period improves slightly by 15.4% a.a. to 18.6% p.a.

When we consider rational rebalancing instead of the calendar criterium, the strategy's expected gross return once again suffers a slight deterioration of 24.3% a.a. to 21.5% pa. However, when we optimize the purchase and sale of shares taking into account the costs, mainly the market impact, the turnover of the strategy is reduced to less than half, as well as the costs and transactions that are reduced by 5.7% to 2.2%. Once again, we see that the average annualized net return improves slightly from 18.6% to 19.3%.

As the strategy's AUM increases, the table indicates that it is more important to progressively limit the exposure of less liquid shares or optimize the rebalancing while take transaction costs into account. With AUM = R $ 3bn, the average net return on the original model of 8.0% a.a. would be virtually equal to the Ibovespa's annualized return of 7.9% in the period, but the rebalancing that observed our methodology of imposing a maximum number of daily volumes in illiquid stocks has an average annualized net return of 14.3% . An optimized partial monthly rebalance would raise this value to 16.6% p.a.

For larger AUM, the rational rebalancing increasingly concentrates purchases and sales in the most liquid stocks that suffer less from market impact costs, but which are still attractive because their high score from our multifactor model. This empowers our systematic model to continue generating backtest net average returns significantly above Ibovespa’s, even for very high AUMs: 14.9% p.a for AUM = R $ 5bn and 12.0% p.a. for AUM = R $ 10bn.

5. References.

[1] N. Torre, "Barra market impact model handbook," BARRA Inc., Berkeley, 1997.

[2] Elia Zarinelli, Michele Treccani, J. Doyne Farmer, and Fabrizio Lillo, "Beyond the Square Root: Evidence for Logarithmic Dependence of Market Impact on Size and Participation Rate" Quantitative Finance, Dec 2014.

[3] Donohue, Christopher, and Kenneth Yip, 2003, Optimal Portfolio Rebalancing with Transaction Costs: Improving on Calendar- or Volatility-Based
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[4] Holden Helge, and Holden Lars, Optimal Rebalancing of Portfolios with Transaction Costs. Stochastics An International Journal of Probability and Stochastic Processes · January 2012

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